

THREE SOLUTIONS FOR A NEW KIRCHHOFF–TYPE PROBLEM

YUE WANG, QI-PING WEI AND HONG-MIN SUO*

Abstract. This article concerns on the existence of multiple solutions for a Kirchhoff-type problem with positive and negative modulus. By applying the variational methods and algebraic analysis, we prove that there exist the only three solutions when the parameter is absolutely small than a constant, only two solutions when the parameter is absolutely equals with the constant and an unique solution when the parameter is absolutely greater than the constant. Moreover, we use the algebraic analysis to calculating the constant with the help of one of the Mountain Pass Lemma, Ekeland variational principle, and Minimax principle.

Mathematics subject classification (2020): 35A15, 35B33, 35J62.

Keywords and phrases: New Kirchhoff-type problem, variational method, algebraic analysis.

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