THREE WEAK SOLUTIONS FOR A DEGENERATE
NONLOCAL SINGULAR SUB–LINEAR PROBLEM

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Abstract. Based on one recent abstract critical point result for differentiable and parametric functionals which was recently proved by Ricceri, we establish the existence of three weak solutions for a class of degenerate nonlocal singular sub-linear problems when the nonlinear term admits some hypotheses on the behavior at infinitely or perturbation property.

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