

GREEN'S FUNCTION FOR A DISCRETE FRACTIONAL BOUNDARY VALUE PROBLEM

JAGAN MOHAN JONNALAGADDA* AND N. S. GOPAL

Abstract. In this article, we deduce the expression and the main properties of the Green's function related to a general nabla fractional difference equation with constant coefficients coupled to Dirichlet conditions. In particular, we prove that such function has constant sign on their set of definition, and also satisfies some additional properties that are fundamental to define a suitable Banach space, where to ensure the existence and uniqueness of solutions of nonlinear problems.

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REFERENCES

- [1] RAVI P. AGARWAL, MARIA MEEHAN AND DONAL O'REGAN, *Fixed point theory and applications*, Cambridge Tracts in Math., 141, Cambridge Univ. Press, Cambridge, 2001.
- [2] K. AHRENDT, L. DE WOLF, L. MAZUROWSKI, K. MITCHELL, T. ROLLING AND D. VECONI, *Initial and Boundary Value Problems for the Caputo Fractional Self-Adjoint Difference Equations*, Enlightenment in Pure Appl. Math. **2**, (2016), no. 1.
- [3] FERHAN M. ATICI AND PAUL W. ELOE, *A transform method in discrete fractional calculus*, Int. J. Difference Equ. **2**, (2007), no. 2, 165–176.
- [4] FERHAN M. ATICI AND PAUL W. ELOE, *Initial value problems in discrete fractional calculus*, Proc. Amer. Math. Soc. **137**, (2009), no. 3, 981–989.
- [5] FERHAN M. ATICI AND PAUL W. ELOE, *Linear systems of fractional nabla difference equations*, Rocky Mountain J. Math. **41**, (2011), no. 2, 353–370.
- [6] FERHAN M. ATICI AND PAUL W. ELOE, *Discrete fractional calculus with the nabla operator*, Electron. J. Qual. Theory Differ. Equ. Special Edition **I**, (2009), no. 3, 12 pp.
- [7] FERHAN M. ATICI AND PAUL W. ELOE, *Two-point boundary value problems for finite fractional difference equations*, J. Difference Equ. Appl. **17**, (2011), no. 4, 445–456.
- [8] FERHAN M. ATICI AND PAUL W. ELOE, *Gronwall's inequality on discrete fractional calculus*, Comput. Math. Appl. **64**, (2012), no. 10, 3193–3200.
- [9] MARTIN BOHNER AND ALLAN PETERSON, *Dynamic equations on time scales. An introduction with applications*, Birkhäuser Boston, Inc., Boston, MA, 2001.
- [10] PAUL W. ELOE AND OUYANG ZI, *Multi-term linear fractional nabla difference equations with constant coefficients*, Int. J. Difference Equ. **10**, (2015), no. 1, 91–106.
- [11] PAUL W. ELOE, JAGANMOHAN JONNALAGADDA AND YOUSSEF RAFFOUL, *The large contraction principle and existence of periodic solutions for infinite delay Volterra difference equations*, Turkish J. Math. **43**, (2019), no. 4, 1988–1999.
- [12] PAUL W. ELOE AND JAGANMOHAN JONNALAGADDA, *Mittag-Leffler stability of systems of fractional nabla difference equations*, Bull. Korean Math. Soc. **56**, (2019), no. 4, 977–992.
- [13] PAUL W. ELOE AND JAGANMOHAN JONNALAGADDA, *Quasilinearization applied to boundary value problems at resonance for Riemann–Liouville fractional differential equations*, Discrete Contin. Dyn. Syst. Ser. S **13**, (2020), no. 10, 2719–2734.

- [14] PAUL W. ELOE AND JAGANMOHAN JONNALAGADDA, *Quasilinearization and boundary value problems for Riemann–Liouville fractional differential equations*, Electron. J. Differ. Equ. **2019**, (2019), paper no. 58, 15 pp.
- [15] YOUSEF GHOLAMI AND KAZEM GHANBARI, *Coupled systems of fractional ∇ - difference boundary value problems*, Differ. Equ. Appl. **8**, (2016), no. 4, 459–470.
- [16] CHRISTOPHER GOODRICH AND ALLAN C. PETERSON, *Discrete fractional calculus*, Springer, Cham, 2015.
- [17] ST. JULIA GOAR, *A Caputo boundary value problem in Nabla fractional calculus*, Thesis (Ph.D.), The University of Nebraska – Lincoln, (2016), 112 pp.
- [18] HENRY L. GRAY AND NIEN FAN ZHANG, *On a new definition of the fractional difference*, Math. Comp. **50**, (1988), no. 182, 513–529.
- [19] JOHNNY HENDERSON AND JEFFREY T. NEUGEBAUER, *Smallest eigenvalues for a fractional difference equation with right focal boundary conditions*, J. Difference Equ. Appl. **23**, (2017), no. 8, 1317–1323.
- [20] JOHNNY HENDERSON, RODICA LUCA, ALEXANDRU TUDORACHE, *Existence and nonexistence of positive solutions to a discrete boundary value problem*, Carpathian J. Math. **33**, (2017), no. 2, 181–190.
- [21] JOHNNY HENDERSON AND RODICA LUCA, *Existence of positive solutions for a system of semipositone coupled discrete boundary value problems*, J. Difference Equ. Appl. **25**, (2019), no. 4, 516–541.
- [22] JOHNNY HENDERSON, *Existence of local solutions for fractional difference equations with Dirichlet boundary conditions*, J. Difference Equ. Appl. **25**, (2019), no. 6, 751–756.
- [23] JOHNNY HENDERSON AND JEFFREY T. NEUGEBAUER, *Existence of local solutions for fractional difference equations with left focal boundary conditions*, Fract. Calc. Appl. Anal. **24**, (2021), no. 1, 324–331.
- [24] AREEBA IKRAM, *Lyapunov inequalities for nabla Caputo boundary value problems*, J. Difference Equ. Appl. **25**, (2019), no. 6, 757–775.
- [25] JAGAN MOHAN JONNALAGADDA, *On two-point Riemann–Liouville type nabla fractional boundary value problems*, Adv. Dyn. Syst. Appl. **13**, (2018), no. 2, 141–166.
- [26] JAGAN MOHAN JONNALAGADDA, *Existence results for solutions of nabla fractional boundary value problems with general boundary conditions*, Adv. Theory Non-linear Anal. Appl. **4**, (2020), 29–42.
- [27] JAGAN MOHAN JONNALAGADDA, *An ordering on Green's function and a Lyapunov-type inequality for a family of nabla fractional boundary value problems*, Fract. Differ. Calc. **9**, (2019), no. 1, 109–124.
- [28] JAGAN MOHAN JONNALAGADDA, *Discrete fractional Lyapunov-type inequalities in nabla sense*, Dyn. Contin. Discrete Impuls. Syst. Ser. A Math. Anal. **27**, (2020), no. 6, 397–419.
- [29] KENNETH S. MILLER AND B. ROSS, *Univalent functions, fractional calculus, and their applications*, Papers from the symposium held at Nihon University, Koriyama, May 1-5, 1988. Edited by H. M. Srivastava and Shigeyoshi Owa. Ellis Horwood Series: Mathematics and its Applications. Ellis Horwood Ltd., Chichester; Halsted Press [John Wiley & Sons, Inc.], New York, 1989.
- [30] D. R. SMART, *Fixed point theorems*, Cambridge Tracts in Math., no. 66, Cambridge Univ. Press, London–New York, 1974.
- [31] AIJUN YANG, LI ZHANG AND JOHNNY HENDERSON, *Comparison of smallest eigenvalues for right focal Atici–Eloe fractional difference equations*, J. Korean Soc. Math. Educ. Ser. B Pure Appl. Math. **24**, (2017), no. 4, 191–200.