

UTILIZING AN INTEGRATING FACTOR TO CONVERT A RIGHT FOCAL BOUNDARY VALUE PROBLEM TO A FIXED POINT PROBLEM

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Abstract. An integrating factor is used to convert a conjugate boundary value problem to a fixed point problem. We conclude with an application illustrating the ease of use in finding an upper solution to a family of boundary value problems that one can apply iteration to in order to solve when the nonlinear term is monotonic.

Mathematics subject classification (2020): 47H10, 34B18.

Keywords and phrases: Fixed point, right focal boundary value problem, integrating factor.

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