

EXISTENCE OF MULTIPLE SOLUTIONS TO A P -KIRCHHOFF PROBLEM

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Abstract. In this paper, the authors study the existence and multiplicity of solutions to a nonlocal p -Kirchhoff-type quasilinear elliptic equation with Dirichlet boundary conditions using a variational approach. They also give some new criteria for the existence of sequences of solutions to the problem. Some recent results are extended and improved. Examples are presented to demonstrate the application of the results.

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