AN EFFICIENT HEAT PROBLEM

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Abstract. By means of fixed point theory we study properties of solutions of a Volterra integral heat equation
\[ x(t) = a(t) - \int_0^t A(t-s)f(s,x(s))ds \]
by first mapping it into
\[ x(t) = z(t) + \int_0^t R(t-s)\left[x(s) - \frac{f(s,x(s))}{J}\right]ds \]
where
\[ z(t) = a(t) - \int_0^t R(t-s)a(s)ds, \]
\( R \) is the resolvent of \( JA \), \( J \) is a large positive number, and \( f \) is bounded.

It turns out that the linear part
\[ x(t) = z(t) + \int_0^t R(t-s)x(s)ds \]
has a unique fixed point which is a uniformly good approximation of a fixed point for the non-linear equation.

The objective is to obtain conditions under which the heat applied by \( a(t) \) concentrates on the solution \( x(t) \).

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REFERENCES