Differential Equations & Applications
Volume 14, Number 2 (2022), 239–246 doi:10.7153/dea-2022-14-16

AN EFFICIENT HEAT PROBLEM

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Abstract. By means of fixed point theory we study properties of solutions of a Volterra integral heat equation

$$x(t) = a(t) - \int_0^t A(t-s)f(s,x(s))ds$$

by first mapping it into

$$x(t) = z(t) + \int_0^t R(t-s)\left[x(s) - \frac{f(s,x(s))}{J}\right]ds$$

where

$$z(t) = a(t) - \int_0^t R(t-s)a(s)ds,$$

R is the resolvent of JA, J is a large positive number, and f is bounded.

It turns out that the linear part

$$x(t) = z(t) + \int_0^t R(t-s)x(s)ds$$

has a unique fixed point which is a uniformly good approximation of a fixed point for the non-linear equation.

The objective is to obtain conditions under which the heat applied by $a(t)$ concentrates on the solution $x(t)$.

Mathematics subject classification (2020): 34A08, 34A12, 45D05, 45G05, 47H10.
Keywords and phrases: Volterra, fixed point theory, transformation.

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