

LYAPUNOV-TYPE INEQUALITIES FOR THIRD ORDER NONLINEAR EQUATIONS

BRIAN BEHRENS AND SOUGATA DHAR*

Abstract. We derive Lyapunov-type inequalities for general third order nonlinear equations involving multiple ψ -Laplacian operators of the form

$$(\psi_2((\psi_1(u'))'))' + q(x)f(u) = 0,$$

where ψ_2 and ψ_1 are odd, increasing functions, ψ_1 is sub-multiplicative and $\frac{1}{\psi_1}$ is convex, and f is a continuous function which satisfies a sign condition. Our results utilize q_+ and q_- , as opposed to $|q|$ which appears in most results in the literature. Additionally, these new inequalities generalize previously obtained results, and the proofs utilize a different technique than most other works in the literature. Furthermore, using the obtained inequalities, we obtain a constraint on the location of the maximum of a solution, properties of oscillatory solutions, and an upper bound for the number of zeroes.

Mathematics subject classification (2020): 34B15, 34L15, 34L30, 26D20.

Keywords and phrases: Lyapunov-type inequality, third-order, nonlinear differential equations, oscillation, convex inequalities.

REFERENCES

- [1] E. A. BISHOP, *Holomorphic completion, analytic continuation, and the interpolation of seminorms*, Ann. of Math. **78** (1963) 468–500.
- [2] R. C. BROWN AND D. B. HINTON, *Opial's inequality and oscillation of second-order equations*, Proc. Amer. Math. Soc. **125** (1997), 1123–1129.
- [3] D. CEKMAK, *On Lyapunov-type inequality for a class of nonlinear systems*, Math. Inq. Appl. **16** (2013), 101–108.
- [4] D. CEKMAK, *Lyapunov-type integral inequalities for certain differential equations*, Appl. Math. Comput. **216** (2010), 368–373.
- [5] S. DHAR, Q. KONG, *Liapunov-type inequalities for third-order half-linear equations and applications to boundary value problems*, Nonlinear Analysis **110** (2014), 170–181.
- [6] S. DHAR AND Q. KONG, *Lyapunov-type inequalities for higher order half-linear differential equations*, Appl. Math. Comput. **273** (2016), 114–124.
- [7] S. DHAR AND Q. KONG, *Lyapunov-type inequalities for third-order linear differential equations*, Math. Inequal. Appl. **19** (2016), 297–312.
- [8] S. DHAR AND Q. KONG, *Lyapunov-type inequalities for odd order linear differential equations*, Elec. J. Diff. Eq., **2016** (2016), no. 243, 1–10.
- [9] A. ELBERT, *A half-linear second order differential equation*, Collo. Math. Soc., **30** (1979), 158–180.
- [10] A. M. FINK, D. F. ST. MARY, *On an inequality of Nehari*, Proc. Amer. Math. Soc., **21** (1969), 640–642.
- [11] J. GUSTAVSSON, L. MALIGRANDA, AND J. PEETRE, *A submultiplicative function*, Indagationes Mathematicae (proceedings), **92** (1989) 435–442.
- [12] B. J. HARRIS AND Q. KONG, *On the oscillation of differential equations with an oscillatory coefficient*, Trans. Amer. Math. Soc. **347** (1995).
- [13] P. HARTMAN, *Ordinary Differential Equations*, Birkhauser, Boston (1982).

- [14] E. HILLE AND R. S. PHILLIPS, *Functional analysis and semi-groups*, Amer. Math. Soc. Colloq. Publ., vol. **31**, American Mathematical Society, Providence, RI, 1957.
- [15] J. KISEL'AK, *Lyapunov-type inequality for third-order half linear differential equations*, Tamkang J. Math. **44** (2013), 351–357.
- [16] M. A. KRASNOSEL'SKII AND YA. B. RUTICKII, *Convex Functions and Orlicz Spaces*, Noordhoff Ltd, Groningen, (1961).
- [17] S. G. KREIN, JU. I. PETUNIN, AND E. M. SEMENOV, *Interpolation of Linear Operators*, American Mathematical Society, (1984).
- [18] M. K. KWONG, *On Lyapunov inequality for dis focality*, J. Math. Anal. Appl. **83** (1981), 486–494.
- [19] A. M. LIAPUNOV, *Probleme general de la stabilite du mouvement*, Ann. Math. Studies **17** (1947), 203–474.
- [20] L. MALIGRANDA, *Indices and interpolation*, Dissertationes Math, **234** (1985), 1–54.
- [21] P. L. DE NAPOLI, J. P. PINASCO, *A Lyapunov Inequality for monotone quasilinear operators*, Differential Integral Equations **18** 10 (2005), 1193–1200.
- [22] N. PARHI AND S. PANIGRAHI, *On Liapunov-type inequality for third-order differential equations*, J. Math. Anal. Appl. **233** (1999), 445–460.
- [23] W. T. PATULA, *On the distance between zeroes*, Proc. Amer. Math. Soc. **52** (1975), 247–251.
- [24] J. SÁNCHEZ AND V. VERGARA, *A Lyapunov-type inequality for a ψ -Laplacian operator*, Nonlinear Analysis **74** (2011), 7071–7077.
- [25] A. WINTNER, *On the non-existence of conjugate points*, Amer. J. Math **73** (1951), 368–380.
- [26] X. YANG, *On inequalities of Lyapunov type*, Applied Mathematics and Computation – AMC **134** (2003), 293–300.
- [27] X. YANG, *On Lyapunov inequality for certain higher-order differential equations*, Appl. Math. Comput. **134**, (2003), 301–317.
- [28] QI-MING ZHANG AND XIAOFEI HE, *Lyapunov-type Inequalities for a class of even-ordered differential equations*, J. Inequal. Appl. **5**, (2012).