

DELTA DERIVATIVES OF THE SOLUTION TO A THIRD-ORDER PARAMETER DEPENDENT BOUNDARY VALUE PROBLEM ON AN ARBITRARY TIME SCALE

WILLIAM M. JENSEN, JEFFREY W. LYONS* AND RICHARD ROBINSON

Abstract. We show that the solution of the third order parameter dependant dynamic boundary value problem $y^{\Delta\Delta\Delta} = f(t, y, y^\Delta, y^{\Delta\Delta}, \lambda)$, $y(t_1) = y_1$, $y(t_2) = y_2$, $y(t_3) = y_3$ on a general time scale may be (delta) differentiated with respect to y_1 , y_2 , y_3 , t_1 , t_2 , t_3 , and λ . We show that the (delta) derivative of the solution solves the third order boundary value problem consisting of either the variational equation (in the dense case), the dynamic analogue (in the scattered case), or a modified variational equation in the parameter case with interesting boundary conditions in all cases.

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