A FOURTH–ORDER ITERATIVE BOUNDARY VALUE PROBLEM WITH LIDSTONE BOUNDARY CONDITIONS

ERIC R. KAUFMANN

Abstract. Let $m \geqslant 2$ and $a > 0$. We consider the existence and uniqueness of solutions to the fourth-order iterative boundary value problem

$$x^{(4)}(t) = f(t, x(t), x^{[2]}(t), \dots, x^{[m]}(t)), \quad -a \leqslant t \leqslant a,$$

with solutions satisfying Lidstone boundary conditions $x(-a) = x(a) = 0$, $x''(-a) = x''(a) = 0$. Here the iterative functions are defined by $x^{[2]}(t) = x(x(t))$ and for $j = 3, \dots m, x^{[j]}(t) = x(x^{[j-1]}(t))$. The main tool employed to establish our results is the Schauder fixed point theorem.

Mathematics subject classification (2020): 34B15, 34K10, 39B05.

Keywords and phrases: Iterative differential equation, Schauder fixed point theorem, Lidstone boundary conditions.

REFERENCES

[1] T. A. BURTON, *Stability by fixed point theory for functional differential equations*, Dover Publications, Inc., Mineola, NY, 2006.

[2] S. CHERAIET, A. BOUAKKAZ, AND R. KHEMIS, *Bounded positive solutions of an iterative three-point boundary-value problem with integral boundary condtions*, J. Appl. Math. Comput., **65**, 1–2 2021, 597–610.

[3] J. M. DAVIS, P. W. ELOE, AND J. HENDERSON, *Triple positive solutions and dependence on higher order derivatives*, J. Math. Anal. Appl., **237**, 2 1999, 710–720.

[4] J. M. DAVIS, P. W. ELOE, AND J. HENDERSON, *Comparison of eigenvalues for discrete Lidstone boundary value problems*, Dynam. Systems Appl., **8**, 3–4 1999, 381–388.

[5] J. M. DAVIS AND J. HENDERSON, *Uniqueness implies existence for fourth-order Lidstone boundary value problems*, PanAmer. Math. J., **8**, 4 1998, 23–35.

[6] E. EDER, *The functional-differential equation $x'(t) = x(x(t))$*, J. Differential Equations, **54**, 3 1984, 390–400.

[7] P. W. ELOE, J. HENDERSON, AND H. B. THOMPSON, *Extremal points for impulsive Lidstone boundary value problems*, Mathematical and Computer Modelling, **32**, 2000, pages 687–698.

[8] P. W. ELOE AND M. N. ISLAM, *Monotone methods and fourth order Lidstone boundary value problems with impulse effects*, Commun. Appl. Anal., **5**, 1 2001, 113–120.

[9] P. W. ELOE, *Nonlinear eigenvalue problems for higher order Lidstone boundary value problems*, Electron. J. Qual. Theory Differ. Equ., **2000**, no. 2, 1–8.

[10] M. FEČKAN, *On a certain type of functional-differential equations*, Math. Slovaca, **43**, 1 (1993), 39–43.

[11] E. R. KAUFMANN, *Existence and uniqueness of solutions for a second-order iterative boundary-value problem*, Electron. J. Differential Equations, **2018**, no. 150, 1–6.

[12] X. P. LIU AND M. JIAN, *Initial value problem for a second order non-autonomous functional-differential iterative equation*, Acta Math. Sinica (Chin. Ser.), **45**, 4 2002, 711–718.

[13] V. R. PETUHOV, *On a boundary value problem*, Trudy Sem. Teor. Differencial. Uravneniǐ s Otklon. Argumentom Univ. Družby Narodov Patrisa Lumumby, **3**, 1965, 252–255.

[14] J. SI AND X. WANG, *Smooth solutions of a nonhomogeneous iterative functional-differential equation with variable coefficients*, J. Math. Anal. Appl., **226**, 2 1998, 377–392.

[15] K. WANG, *On the equation* $x'(t) = f(x(x(t)))$, Funkcial. Ekvac., **33**, 3 1990, 405–425.

[16] P. J. Y. WONG, *Triple solutions of complementary Lidstone boundary value problems via fixed point theorems*, Bound. Value Probl., **2014**, 2014:125, 21 pp.

[17] H. Y. ZHAO AND J. LIU, *Periodic solutions of an iterative functional differential equation with variable coefficients*, Math. Methods Appl. Sci., **40**, 1 2017, 286–292.

[18] H. ZHAO, *Smooth solutions of a class of iterative functional differential equations*, Abstr. Appl. Anal., **2012**, Art. ID 954352, 13 pp.