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SPECTRAL CHARACTERIZATION OF THE CONSTANT SIGN GREEN'S FUNCTIONS FOR PERIODIC AND NEUMANN BOUNDARY VALUE PROBLEMS OF EVEN ORDER

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Abstract. In this paper we will characterize the interval of real parameters M in which the Green's function G_M, related to the operator $T_{2n}[M]u(t) := u^{(2n)}(t) + Mu(t)$ coupled to periodic, $u^{(i)}(0) = u^{(i)}(T)$, $i = 0,\dots,2n-1$, or Neumann, $u^{(2i+1)}(0) = u^{(2i+1)}(T) = 0$, $i = 0,\dots,n-1$, boundary conditions, has constant sign on its square of definition. More concisely, we will prove that the optimal values are given as the first zeros of $G_M(0,0)$ or $G_M(T/2,0)$, depending both on the sign of G_M and on the fact whether $2n$ is, or is not, a multiple of 4. Such values will be characterized as the eigenvalues of the operator $u^{(2n)}$ related to suitable boundary conditions. This characterization allows us to obtain such values without calculating the exact expression of the Green's function.

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