

BOUNDARY VALUE PROBLEM FOR HYBRID GENERALIZED HILFER FRACTIONAL DIFFERENTIAL EQUATIONS

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Abstract. This manuscript is concerned with the existence of solutions for a class of boundary value problems for nonlinear fractional hybrid differential equations involving generalized Hilfer fractional derivative. The main result is based on a fixed point theorem due to Dhage, which is illustrated with examples.

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