ON THE STABILITY OF A VISCOELASTIC TIMOSHENKO SYSTEM WITH MAXWELL–CATTANEO HEAT CONDUCTION

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Abstract. This paper discusses a thermoelastic Timoshenko system with viscoelastic damping acting on the shear force, and heat conduction given via Maxwell-Cattaneo's law (usually called second sound) on the bending moment. We establish a general decay estimate for the solution energy. The exponential and polynomial decay results are only special cases of the present work. The obtained result shows that the viscoelastic damping on the shear force and the thermal damping on the bending moment are strong enough to stabilize the system without any additional restrictions like "the equal-wave of speed propagation" or "the stability number" conditions which are usually associated with similar problems.

Mathematics subject classification (2020): 35B35, 35D30, 74D10, 74F05, 74J30.
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