

## EXISTENCE OF SOLUTIONS FOR NONLOCAL ELLIPTIC SYSTEMS WITH EXPONENTIAL NONLINEARITY

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*Abstract.* In this paper, we establish the existence of solutions for a Kirchhoff-type system with Dirichlet boundary condition and nonlinearities having exponential critical growth. Our approach is based on the Trudinger-Moser inequality and on a minimax theorem.

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### REFERENCES

- [1] ADIMURTHI, *Existence of Positive Solutions of the Semilinear Dirichlet Problem with Critical Growth for the  $n$ -Laplacian*, Ann. Della. Scuola. Norm. Sup. di Pisa, Serie **4**, tome 17, 3 (1990), 393–413.
- [2] A. AMBROSETTI, H. BREZIS AND G. CERAMI, *Combined effects of concave and convex nonlinearities in some elliptic problems*, J. Funct. Anal., **122**, 2 (1994), 519–543.
- [3] N. T. CHUNG, *Multiplicity results for a class of  $p(x)$ -Kirchhoff type equations with combined nonlinearities*, E. J. Qualit. Theory Diff. Equ., **42**, (2012), 1–13.
- [4] N. T. CHUNG, *An existence result for a class of Kirchhoff type systems via sub and supersolutions method*, Appl. Math. Lett., **35** (2014), 95–101.
- [5] J. L. CHERN, Z. Y. CHEN AND C. S. LIN, *Uniqueness of Topological Solutions and the Structure of Solutions for the Chern-Simons System with Two Higgs Particles*, Commun. Math. Phys., **296**, (2010), 323–351.
- [6] D. G. DE FIGUEIREDO, O. H. MIYAGAKI AND B. RUF, *Elliptic equations in  $\mathbb{R}^2$  with nonlinearities in the critical growth range*, Calc. Var. Partial Differential Equations, **3**, 2 (1995) 139–153.
- [7] D. G. DE FIGUEIREDO, J. M. DO Ó, AND B. RUF, *Critical and subcritical elliptic systems in dimension two*, Indiana Univ. Math. J., **53**, 4 (2004), 1037–1054.
- [8] G. M. FIGUEIREDO AND U. B. SEVERO, *Ground state solution for a Kirchhoff problem with exponential critical growth*, Milan J. Math., **84** (2015), 23–39.
- [9] N. GHOUSSEB AND D. PREISS, *A general mountain pass principle for locating and classifying critical points*, Anal. Non Lineaire, **6**, 5 (1989), 321–330.
- [10] X. HAN AND G. DAI, *On the sub-supersolution method for  $p(x)$ -Kirchhoff type equations*, J. Inequal. Appl., **283** (2012) 1–11.
- [11] B. JOAO MARCOS DO O, *Semilinear Dirichlet problems for the  $N$ -Laplacian in  $\mathbb{R}^N$  with nonlinearities in critical growth range*, Differential and Integral equations, **9**, 5 (1996), 967–979.
- [12] G. KIRCHHOFF, *Mechanik*, Teubner, Leipzig, Germany 1883.
- [13] Y. LI, P. LIU, AND Y. YANG, *Moser-Trudinger inequalities of vector bundle over a compact Riemannian manifold of dimension 2*, Calc. Var., **28**, (2006), 59–83.
- [14] C. S. LIN, A. C. PONCE, AND Y. YANG, *A system of elliptic equations arising in Chern-Simons field theory*, J. Fun. Anal., **247** 2 (2007), 289–350.
- [15] C. S. LIN, AND J. V. PRAJAPAT, *Vortex Condensates for Relativistic Abelian Chern-Simons Model with Two Higgs Scalar Fields and Two Gauge Fields on a Torus*, Commun. Math. Phys., **288**, (2009), 311–347.
- [16] C. S. LIN, AND S. YAN, *Bubbling Solutions for Relativistic Abelian Chern-Simons Model on a Torus*, Commun. Math. Phys., **297**, 3 (2010), 733–758.

- [17] J. MOSER, *A sharp form of an inequality by N. Trudinger*, Indiana Univ. Math. Jour., **20**, 11 (1971), 1077–1092.
- [18] N. MEGREZ, K. SREENADH, B. KHALDI, *Multiplicity of positive solutions for a gradient system with an exponential nonlinearity*, EJDE, **236** (2012), 1–16.
- [19] O. H. MIYAGAKI AND R. S. RODRIGUES, *On positive solutions for a class of singular quasilinear elliptic systems*, J. Math. Anal. Appl., **334** 2 (2007), 818–833.
- [20] S. PRASHANTH AND K. SREENADH, *Multiplicity results in a ball for  $p$ -Laplace equation with positive nonlinearity*, Advances in Differential Equations, **7**, 7(2002), 877–896.
- [21] S. PRASHANTH AND K. SREENADH, *Existence of multiple positive solutions for  $N$ -Laplacian in a bounded domain in  $\mathbb{R}^N$* , Advanced Nonlinear Studies, **5**, 1 (2005), 13–21.
- [22] N. S. TRUDINGER, *On embedding into Orlicz spaces and some applications*, Journal of Math. Mech., **17**, 5 (1967), 473–484.