

ON THE PERIODIC SOLUTIONS FOR NONLINEAR VOLTERRA–FREDHOLM INTEGRO–DIFFERENTIAL EQUATIONS WITH ψ -HILFER FRACTIONAL DERIVATIVE

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Abstract. In this research paper, we present some results about the existence and uniqueness of periodic solutions for a great nonlinear class of Volterra-Fredholm integro-differential equations equipped with fractional integral conditions, involving ψ -Hilfer fractional operator. This investigation is carried out by means of the coincidence degree theory of Mawhin. A typical example is also presented.

Mathematics subject classification (2020): 34A08, 34A12, 34B40, 45J05.

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