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GLOBAL CONVERGENCE OF RANK–ONE PGD APPROXIMATIONS BY ALTERNATE MINIMIZATION

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Abstract. Low-rank tensor approximations of solutions to high dimensional partial differential problems have shown their great relevance among the most used numerical methods in recent years, both in terms of accuracy and time computation. The central point of these methods is the computation of an optimal low-rank tensor to enrich, in a progressive way, the obtained tensorial approximation. For minimization problems, this point can be performed through the classical alternate minimization method. However, the transition to the tensorial framework breaks the linearity and convexity of the considered problems and their associated functionals, which impacts the convergence of the alternate minimization sequences. In the literature, only local convergence results and global convergence results, under some restrictive hypotheses, are available.

In the following work, we give an unconditional convergence result of the alternate minimization scheme to compute the optimal low-rank tensor, for multi-dimensional variational linear elliptic equations. Also, we provide an adequate choice of the initialization as well as a relevant stopping criterion in the alternating minimization process.

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