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EXISTENCE AND NONEXISTENCE OF SOLUTIONS OF THIN–FILM EQUATIONS WITH VARIABLE EXPONENT SPACES

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Abstract. The paper aims at presenting a thin film problem involving variable exponent sources in a bounded domain. For the problem, we give attention to the existence and nonexistence of solutions under subcritical initial energy. We determine the global existence of solutions, exponential decay and blow-up of solutions in finite time with specific conditions for the proposed model.

Mathematics subject classification (2020): 35B44, 35D30, 35K70.
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