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MULTIPLE SOLUTIONS FOR NONLOCAL FRACTIONAL KIRCHHOFF TYPE PROBLEMS

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Abstract. In this paper, using variational methods and critical point theory, we establish the existence of two and infinitely many solutions for a fractional Kirchhoff type problem driven by a nonlocal operator of elliptic type in a fractional Orlicz-Sobolev space with homogeneous Dirichlet boundary conditions. Some examples are presented to demonstrate the application of our main results.

Mathematics subject classification (2020): 35B38, 47A75, 34L30.
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