ANALYSIS AND CONTROL OF PHYSIOLOGICALLY
STRUCTURED MODELS WITH NONLOCAL DIFFUSION

SIDI MOHAMMED BOUGUIMA* AND KHADIDJA AICHA KADA

Abstract. In the first part of this paper, a non-autonomous physiologically structured model with nonlocal diffusion is developed. Using the semigroup generated by the diffusion operator and characteristic method, the problem is reformulated as a fixed point problem in a suitable Banach space. We give conditions under which the model admits a unique positive solution. In the second part of this work, we give an application of the study done in the first part. We consider an optimal control problem. The optimal strategies are discussed using normal cone and dual techniques.

Mathematics subject classification (2020): 35L02, 35L45, 35L50, 49K20, 92D25, 92D40.
Keywords and phrases: Physiologically structured model, nonlocal diffusion, semigroup, fixed point, optimal control.

REFERENCES

[1] A. ACKLEH, K. DENG, AND X. WANG, *Competitive exclusion and coexistence for a quasilinear size-structured population model*, Mathematical Biosciences **192**, (2004), 177–192.

[2] B. AINSEBA, S. M. BOUGUIMA, *An adaptative model for a multistage structured population under fluctuation environment*, Discrete Contin. Dyn. Syst. Ser. B. **25**, 6 (2020), 2331–2349.

[3] H. AMANN, *Ordinary Differential Equations, An introduction to nonlinear Analysis*, De Gruter Studies in Mathematics **13**, Berlin, NY, 1990.

[4] F. ANDREU-VAILLO, J. M. MAZON, J. D. ROSSI, J. TOLEDO-MELERO, *Nonlocal Diffusion Problems*, Mathematical Surveys and Monographs, AMS, Providence, Rhode Island, 2010.

[5] S. ANITA, *Analysis and Control of Age-Dependent Population Dynamics*, Kluwer, Dordrecht, Netherlands, 2000.

[6] S. ANITA, V. CAPASSO, AND A. M. MOSNEAGU, *Regional control in optimal harvesting problems of population dynamics*, Nonlin. Anal., **147**, (2016), 191–212.

[7] R. ANGUELOV, C. DUFOURD, Y. DUMONT, *Mathematical model for pest-insect control using mating disruption and trapping*, Appl. Math. Model. **52** (2017), 437–457.

[8] J. P. AUBIN, I. EKELAND, *Applied Nonlinear Analysis*, John Wiley and Sons, New York, USA, 1984.

[9] V. BARBU, M. IANNELLI, *Controlling the SIS age structured epidemics*, in: Mathematical Model and Health Sciences, Vanderbilt (1998), 29–34.

[10] P. W. BATES, *On some nonlocal evolution equations arising in materials science*, in: Brunner, H., Zhao, X.-Q., Zou, X. (eds.) Nonlinear Dynamics and Evolution Equations, Fields Institute Communications, vol. **48**, 13–52, American Mathematical Society, Providence, 2006.

[11] P. BATES, P. FIFE, X. REN, X. WANG, *Travelling waves in a convolution model for phase transitions*, Arch. Ration. Mech. Anal. **138**, (1997), 105–136.

[12] A. O. BELYAKOV, AND V. M. VELIOV, *On optimal harvesting in age-structured populations*, in Dynamic Perspectives on Managerial Decision Making (H. Dawid, K. F. Doerner, G. Feichtinger, P. M. Kort, A. Seidl, Eds.), Springer Internat. Publ. **22**, (2016), 149–166.

[13] M. A. BOGOYA, *nonlocal nonlinear diffusion equation in higher space dimensions*, J. Math. Anal. Appl. **344**, (2008), 601–615.

[14] F. BRAUER, *Nonlinear age dependent population growth under harvesting*, Comput. Math. Appl. **9**, (1983), 345–352.

[15] M. BROKATE, *Pontryagin's principle for control problems in age-dependent population dynamics*, Journal of Mathematical Biology **23** 1 (1985), 75–101.

[16] G. BURSONI, S. A. MATUCCI, *problem of optimal harvesting policy in two stage age dependent population model*, Math. Biosci. **143**, (1997), 1–33.

[17] R. BURGER, *The mathematical theory of selection, recombination, and mutation*, Wiley series in mathematical and computational biology, John Wiley, 2000.

[18] A. CALSINA, AND J. A. SALDANA, *Model of physiologically structured population dynamics with nonlinear growth structured population dynamics with nonlinear individual growth rate*, Journal of Mathematical Biology **33**, (1995), 335–364.

[19] N. CHAMPAGNAT, R. FERRIERE, AND S. MELEARD, *Individual-based probabilistic models of adaptative evolution and various scaling approximations*, Seminar on Stochastic Analysis, Random Fields and Applications (R. C. Dalang, F. Russo, and M. Dzzi, eds.), Progress in Probability, vol. **59**, Birkhäuser Basel, (2008), 75–113.

[20] J. Z. FARKAS, AND P. HINOW, *Physiologically structured populations with diffusion and dynamic boundary conditions*, Mathematical Biosciences and Engineering **8**, (2011), 503–513.

[21] N. HRITONENKO, AND Y. YATSENKO, *Optimization of harvesting age in integral age-dependent model of population dynamics*, Math. Biosci. **195**, (2005), 154–167.

[22] V. HUTSON, W. SHEN, AND G. T. VICKERS, *Spectral theory for nonlocal dispersal with periodic or almost-periodic time dependence*, Rocky Moutain Journal of Mathematics **38**, (2008), 1147–1175.

[23] M. IANNELLI, AND F. MILNER, *The Basic Approach to Age-Structured Population Dynamics. Models, Methods and Numerics*, Lecture Notes on Mathematical Modelling in the Life Sciences, Springer, Dordrecht, 2017.

[24] H. KANG, S. RUAN, *Nonlinear age-structured population models with nonlocal diffusion and nonlocal boundary conditions*, Journal of Differential Equations **278**, (2021), 430–462.

[25] H. KANG, S. RUAN, AND X. YU, *Age-structured population dynamics with nonlocal diffusion. Journal of Dynamics and Differential Equations*, (2020), https://doi.org/10.1007/s10884-020-09860-5.

[26] N. KATO, *Positive global solution for a general model of size-dependent populations dynamics*, Abstr. Appl. Anal., **5**, 3 (2000), 191–206.

[27] E. B. LEE, AND L. MARKUS, *Foundations of Optimal Control Theory*, John Wiley and Sons, New York, 1967.

[28] S. LENHART, AND J. T. WORKMAN, *Optimal Control Applied to Biological Models*, Chapman and Hall, 2007.

[29] J. LIUA, X. S. WANG, *Numerical optimal control of a size-structured PDE model for metastatic cancer treatment*, Mathematical Biosciences **314**, (2019), 28–42.

[30] R. LIU, G. LIU, *Optimal birth control problems for a nonlinear vermin population model with size-structure*, J. Math. Anal. Appl. **449**, (2017), 265–291.

[31] J. LIUA, X. S. WANG, *Numerical optimal control of a size-structured PDE model for metastatic cancer treatment*, Mathematical Biosciences **314**, (2019), 28–42.

[32] Z. LUO, *Optimal harvesting problem for an age-dependent n-dimensional food chain diffusion model*, Appl. Math. Comput. **186**, (2007), 1742–1752.

[33] H. LONG-GUANG, Z. XIAN, *Cone metric spaces and fixed point theorems of contractive mappings*, J. Math. Anal. Appl. **332**, (2007), 1468–1476.

[34] D. MAITY, M. TUCSNAK, E. ZUAZUA, *Controllability and positivity constraints in population dynamics with age structuring and diffusion*, J. Math. Pures Appl.Vol. **129**, (2019), 153–179.

[35] P. MAGAL, S. RUAN, *Structured-Population Models in Biology and Epidemiology*, Springer, Berlin, 2008.

[36] A. MALIK, N. SINGH, S. SATYA, *House fly (Musca domestica): a review of control strategies for a challenging pest*, J. Environ. Sci. Health **42**, 4 (2007), 453–69.

[37] M. J. METZ, AND O. DIEKMANN, *The dynamics of physiologically structured populations*, vol. **68** of Lectures Notes in Biomathematics, Berlin, Springer-Verlag, 1986.

[38] L. F. MURPHY, S. J. SMITH, *Optimal Harvesting of an age structured population*, J. Math. Biology **29**, (1990), 77–90.

[39] D. PICART, B. AINSEBA, F. MILNER, *Optimal control problem on insect pest populations*, Applied Mathematics Letters **24**, (2011), 1160–1164.

[40] J. W. SINKO, AND W. STREIFER, *A new model of age-size structure of a population*, Ecology **48**, (1967), 910–918.

[41] V. A. VASSILIOU, *Control of Lobesia botrana (Lepidoptera: Tortricidae) in vineyards in Cyprus using the mating disruption technique, Crop Protection*, **28**, (2009), 145–150.

[42] G. F. WEBB, *Theory of Nonlinear Age-Dependent Populations Dynamics*, Monographs Textbooks in Pure and Applied Math. Dekker, NY 1985.

[43] G. F. WEBB, *Population models structured by age, size, and spatial position*, in: Magal, P., Ruan, S. (eds.) Structured Population Models in Biology and Epidemiology. Lecture Notes Mathematics, vol. **1936**, pp. 1–49. Springer, Berlin, 2008.

[44] F. Y. YANG, W. T. LI, AND S. RUAN, *Dynamics of a nonlocal dispersal SIS epidemic model with Neumann boundary conditions*, J. Differential Equations, **267**, 3 (2019), 2011–2051.

Differential Equations & Applications
www.ele-math.com
dea@ele-math.com

Differential Equations & Applications
www.ele-math.com
dea@ele-math.com