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EXTREMAL SOLUTIONS AT INFINITY FOR SYMPLECTIC SYSTEMS ON TIME SCALES II — EXISTENCE THEORY AND LIMIT PROPERTIES

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Abstract. In this paper we continue with our investigation of principal and antiprincipal solutions at infinity solutions of a dynamic symplectic system. The paper is a continuation of part I appeared in Differential Equations and Applications in 2022, where we have presenteded a theory of genera of conjoined bases for symplectic dynamic systems on time scales and its connections with principal solutions at infinity and antiprincipal solutions at infinity for these systems together with some basic properties of this new concept on time scales. Here we provide a characterization of all principal solutions of dynamic symplectic system at infinity in the given genus in terms of the initial conditions and a fixed principal solution at infinity from this genus. Further, we provide a characterization of all antiprincipal solutions of dynamic symplectic system at infinity in the given genus in terms of the initial conditions and a fixed principal solution at infinity from this genus. We also establish mutual limit properties of principal and antiprincipal solutions at infinity.

Mathematics subject classification (2020): Primary 34N05; Secondary 34C10, 39A12, 39A21.

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