

IMPLICIT CAPUTO FRACTIONAL q -DIFFERENCE EQUATIONS WITH NON INSTANTANEOUS IMPULSES

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Abstract. In the present article, we prove some existence results for a class of implicit Caputo fractional q -difference equations with non instantaneous impulses in Banach spaces. The used techniques rely on the concepts of measure of noncompactness and the use of suitable fixed point theorems.

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