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UNIQUE SOLVABILITY OF SECOND ORDER NONLINEAR TOTALLY CHARACTERISTIC EQUATIONS

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Abstract. We consider a second order singular nonlinear partial differential equation of the form $(t\partial_t)^2 u = F(t,x,u,\partial_x u,\partial_x^2 u,t\partial_t u,t\partial_t\partial_x u)$, where F is assumed to be continuous in t and holomorphic with respect to the other variables. Under certain conditions, we prove that the equation has a unique solution that is continuous in t and holomorphic in x.

Mathematics subject classification (2020): 35A01, 35A02, 35A10, 35G20.
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