

## EFFECTS OF RAPID POPULATION GROWTH ON TOTAL BIOMASS IN MULTI-PATCH ENVIRONMENT

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*Abstract.* In this work, we study a multi-patch model, where the patches are coupled by asymmetrical migration terms, and each patch follows a logistic law under the assumption that some growth rates are much larger than the other. First, for Two-patch model where one growth rate is much larger than the second one, the total equilibrium population is greater or smaller than the sum of two carrying capacities for all migration rates. Second, we consider Three-patch model in the two cases: (i) where two growth rates are much larger than the third one, and (ii) where one growth rate is much larger than the other two. For both cases, we give a complete classification of all possible situations under which the fragmentation can lead to a total equilibrium population greater or smaller than the sum of the three carrying capacities. Finally, in the general case, we consider the model of  $n$  patches with the assumption that: (i) all growth rates but one are much larger than the  $n$ th growth rate, (ii) two blocks where the growth rates of the first block are much larger than that of the second one. For the first case, we give a complete classification of all possible situations under which the fragmentation can lead to a total equilibrium population greater or smaller than the sum of the  $n$  carrying capacities, and in the second case, we construct a reduced model and we prove its global stability.

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