# CANONICAL REPRESENTATION OF THIRD–ORDER DELAY DYNAMIC EQUATIONS ON TIME SCALES

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*Abstract.* The authors show that any semicanonical or noncanonical third-order linear delay dynamic equation on a time scale can be written in canonical form without imposing any additional conditions on the coefficient functions. Since this is true for any time scale, this means it holds for differential and difference equations. The implication of this is the significant result that any set of conditions which show that a related equation in canonical form is oscillatory will guarantee that the semicanonical or noncanonical equation is also oscillatory. Several examples of the application of the results are incorporated into the paper.

*Mathematics subject classification* (2020): 34C10, 34K11, 34N05.

*Keywords and phrases*: Third-order delay dynamic equation, time scale, canonical representation, oscillation.

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