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QUALITATIVE ANALYSIS OF NEUTRAL IMPLICIT FRACTIONAL q–DIFFERENCE EQUATIONS WITH DELAY

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Abstract. This paper explores the existence and stability of implicit neutral Caputo fractional q-difference equations within four distinct classes, incorporating various delay types such as finite, infinite, and state-dependent delays. To establish the existence of solutions, we utilize the fixed point theorem of Krasnoselskii in Banach spaces. The concluding section provides illustrative examples that highlight the obtained results.

Mathematics subject classification (2020): 26A33, 34K37.
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