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BLOW–UP SOLUTIONS FOR NON–SCALE–INVARIANT NONLINEAR SCHRÖDINGER EQUATION IN ONE DIMENSION

MASARU HAMANO*, MASAHIRO IKEDA AND SHUJI MACHIHARA

Abstract. In this paper, we consider the mass-critical nonlinear Schrödinger equation in one dimension. Ogawa–Tsutsumi [Proc. Amer. Math. Soc. **111** (1991), no. 2, 487–496] proved a blow-up result for negative energy solution by using a scaling argument for initial data. In general, a equation with a linear potential does not have a scale invariant, so the method by Ogawa–Tsutsumi cannot be used directly to that. In this paper, we prove a blow-up result for the equation with the linear potential by modifying the argument of Ogawa–Tsutsumi.

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