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RESULTS ON NON–INSTANTANEOUS IMPULSIVE φ–CAPUTO FRACTIONAL DIFFERENTIAL SYSTEMS: STABILITY AND CONTROLLABILITY

RAJESH DHAYAL*, MUSLIM MALIK AND KOTTAKKARAN SOOPPY NISAR

Abstract. The main objective of this work is to investigate a class of φ-Caputo fractional differential systems with impulsive effects and nonlocal conditions. We used Banach fixed point theorem, fractional calculus, and semigroup theory to study the existence of piecewise continuous mild solution for the proposed system. Moreover, we proved the novel stability criteria for the considered system. Further, we investigated the exact and trajectory controllability of the proposed system. Finally, the main results are validated with the aid of an example.

Mathematics subject classification (2020): 34G20, 34A37, 34A08, 93B05.
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