Differential
Equations
& Applications
Volume 16, Number 2 (2024), 135–150
doi:10.7153/dea-2024-16-08

EXISTENCE AND MULTIPLICITY OF SOLUTIONS FOR $p(x)$–KIRCHHOFF TYPE PROBLEMS WITH NONHOMOGENEOUS NEUMANN CONDITIONS

FARIBA GHAREHGAZLOUEI AND SHAPOUR HEIDARKHANI*

Abstract. In this paper, we are interested to discuss the existence of multiple solutions for a class of $p(x)$-Kirchhoff type equations with nonhomogeneous Neumann boundary conditions arising in modelling of various phenomena in the study of nonlinear elasticity theory, electro-rheological fluids, and so on. By using a consequence of the local minimum theorem due to Bonanno we look into the existence of one solution under algebraic conditions on the nonlinear term, and two solutions for the problem under algebraic conditions with the classical Ambrosetti-Rabinowitz condition on the nonlinear term. Furthermore, by employing a three-critical-point theorem due to Bonanno and Marano, we guarantee the existence of three solutions for the problem in a special case.

Mathematics subject classification (2020): 35J20, 35J60.
Keywords and phrases: $p(x)$-Kirchhoff type equation, variable exponent Sobolev spaces, weak solutions, variational principle.

REFERENCES

[1] G. BONANNO, *A critical point theorem via the Ekeland variational principle*, Nonlinear Anal. TMA **75**, (2012), 2992–3007.
[2] G. BONANNO, *Relations between the mountain pass theorem and local minima*, Adv. Nonlinear Anal. **1**, (2012), 205–220.
[3] G. BONANNO AND P. CANDITO, *Three solutions to a Neumann problem for elliptic equations involving the p-Laplacian*, Arch. Math. (Basel) **80**, (2003), 424–429.
[4] G. BONANNO AND A. CHINNÌ, *Existence and multiplicity of weak solutions for elliptic Dirichlet problems with variable exponent*, J. Math. Anal. Appl. **418**, (2014), 812–827.
[5] G. BONANNO AND S. A. MARANO, *On the structure of the critical set of non-differentiable functions with a weak compactness condition*, Appl. Anal. **89**, (2010), 1–10.
[6] M. M. BOUREANU, A. MATEI AND M. SOFONEA, *Nonlinear problems with $p(.)$-growth conditions and applications to antiplane contact models*, Adv. Nonlinear Stud. **14**, 2 (2014), 295–313.
[7] F. CAMMAROTO AND L. VILASI, *Multiple solutions for a Kirchhoff-type problem involving the $p(x)$-Laplacian operator*, Nonlinear Anal. **74**, (2011), 1841–1852.
[8] F. CAMMAROTO AND L. VILASI, *Sequences of weak solutions for a Navier problem driven by the $p(x)$-biharmonic operator*, Minimax Theory and its Applications **4**, (2019), 71–85.
[9] Y. CHEN, S. LEVINE AND R. RAO, *Variable exponent, linear growth functionals in image restoration*, SIAM Journal of Applied Mathematics **66**, (2006), 1383–1406.
[10] N. T. CHUNG, *Multiplicity results for a class of $p(x)$-Kirchhoff type equations with combined nonlinearities*, Electron. J. Qual. Theory Differ. Equ. **2012**, (2012), 1–13.
[11] G. D'AGUÌ AND A. SCIAMMETTA, *Infinitely many solutions to elliptic problems with variable exponent and nonhomogeneous Neumann conditions*, Nonlinear Anal. **75**, (2012), 5612–5619.
[12] G. DAI AND J. WEI, *Infinitely many non-negative solutions for a $p(x)$-Kirchhoff-type problem with Dirichlet boundary condition*, Nonlinear Anal. **73**, (2010), 3420–3430.

Paper DEA-16-08

[13] A. L. A. DE ARAUJO, S. HEIDARKHANI, G. A. AFROUZI AND S. MORADI, *A variational approach for nonlocal problems with variable exponent and nonhomogeneous Neumann conditions*, Annals of the University of Craiova, Mathematics and Computer Science Series **48**, 2 (2021), 206–221.
[14] S. G. DENG, *A local mountain pass theorem and applications to a double perturbed $p(x)$-Laplacian equations*, Appl. Math. Comput. **211**, (2009), 234–241.
[15] L. DIENING, P. HARJULEHTO, P. HÄSTÖ AND M. RŮŽIČKA, *Lebesgue and Sobolev Spaces with Variable Exponents*, in: Lecture Notes in Mathematics, vol. 2017, Springer-Verlag, Heidelberg, 2011.
[16] L. C. EVANS, *Partial Differential Equations*, American Mathematical society, Providence, Rhode Island, 1998.
[17] X. L. FAN AND C. JI, *Existence of infinitely many solutions for a Neumann problem involving the $p(x)$-Laplacian*, J. Math. Anal. Appl. **334**, (2007), 248–260.
[18] X. L. FAN AND D. ZHAO, *On the spaces $L^{p(x)}(\Omega)$ and $W^{m,p(x)}(\Omega)$*, J. Math. Anal. **263**, (2001), 424–446.
[19] S. HEIDARKHANI, A. L. A. DE ARAUJO, G. A. AFROUZI AND S. MORADI, *Existence of three weak solutions for Kirchhoff-type problems with variable exponent and nonhomogeneous Neumann conditions*, Fixed Point Theory **21** (2020), 525–548.
[20] S. HEIDARKHANI, A. L. A. DE ARAUJO, G. A. AFROUZI AND S. MORADI, *Multiple solutions for Kirchhoff-type problems with variable exponent and nonhomogeneous Neumann conditions*, Math. Nach. **291**, (2018), 326–342.
[21] S. HEIDARKHANI, A. L. A. DE ARAUJO, G. CARISTI AND A. SALARI, *Multiplicity results for non-local problems with variable exponent and nonhomogeneous Neumann conditions*, Dynamic Systems and Applications **30**, 7 (2021) 1149–1179.
[22] M. HSSINI, M. MASSAR AND N. TSOULI, *Existence and multiplicity of solutions for a $p(x)$-Kirchhoff type problems*, Bol. Soc. Parana. Mat. **33**, (2015), 201–215.
[23] S. KICHENASSAMY AND L. VERON, *Singular solutions of the p-Laplace equation*, Math. Ann. **275**, (1986), 599–615.
[24] G. KIRCHHOFF, *Vorlesungen über mathematische Physik: Mechanik*, Teubner, Leipzig, 1883.
[25] O. KOVÁČÍK AND J. RÁKOSNÍK, *On the spaces $L^{p(x)}$ and $W^{1,p(x)}$*, Czechoslovak Math. **41**, (1991), 592–618.
[26] M. RŮŽIČKA, *Electrorheological Fluids: Modeling and Mathematical Theory*, Springer-Verlag, Berlin, (2002).
[27] L. VILASI, *A non-homogeneous elliptic problem in low dimensions with three symmetric solutions*, J. Math. Anal. Appl. **501**, (2021), 124074.
[28] E. ZEIDLER, *Nonlinear functional analysis and its applications*, Vol. II/B, Springer-Verlag, New York, 1990.
[29] V. V. ZHIKOV, *Averaging of functionals in the calculus of variations and elasticity*, Math. USSR Izv. **29**, (1987), 33–66.