SOLVABILITY FOR A COUPLED SYSTEM OF 4–SEQUENTIAL FRACTIONAL DIFFERENTIAL EQUATIONS

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Abstract. The present work deals with a coupled system of fractional differential equations involving four sequential Caputo derivatives in each of its components. The fractional differential system gives rise to a standard coupled system of two ordinary differential equations of order four, which has practical applications in some real-world phenomena such as robotics, aerospace, and electrical engineering. The existence of a unique vector solution for our sequential system is studied. The existence of at least one vector solution for the considered system is also investigated. Some illustrative examples are discussed in detail to show the main results' applicability. The stabilities in the sense of Ulam Hyers for the system is discussed. A conclusion follows at the end.

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