

EXISTENCE OF SOLUTION FOR HIGHER ORDER NONLINEAR CAPUTO FRACTIONAL DIFFERENTIAL EQUATION WITH NONLINEAR GROWTH

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Abstract. This research paper explores the existence of solution to a higher-order fractional differential equation with a general boundary condition, shedding light on novel extensions beyond existing literature. The equation, characterized by a Caputo fractional derivative exhibits non-linearity and resonance, making it a compelling subject of study. The investigation employs coincidence degree theory, a robust tool for the examination of differential equations and the identification of solution. Notably, this paper delves into nonlinear growth patterns of function. The main results of the research are accompanied by an illustrative example to clarify the concepts discussed.

Mathematics subject classification (2020): 26A33, 34B10, 34B15, 34A08, 34B18.

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