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ON AN INVERSE BOUNDARY–VALUE PROBLEM FOR THE EQUATION OF MOTION OF A HOMOGENEOUS ELASTIC BEAM WITH PINNED ENDS

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Abstract. This paper is devoted to the study of the inverse boundary-value problem for the linearized equation of motion of a homogeneous beam with pinned ends. The primary goal of the work is to study the existence and uniqueness of the classical solution of the considered inverse boundary-value problem. To investigate the solvability of the considered problem, we carried out a transformation from the original problem to some auxiliary equivalent problem with trivial boundary conditions. Furthermore, we prove the existence and uniqueness theorem for the auxiliary problem by the contraction mappings principle. Based on the equivalency of these problems is shown the existence and uniqueness of the classical solution of the original problem.

Mathematics subject classification (2020): Primary 35R30, 35L25, 35Q74; Secondary 35A01, 35A02, 35A09.

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