Differential
Equations
& Applications
Volume 16, Number 4 (2024), 279–304
doi:10.7153/dea-2024-16-17

ON THE IMPULSIVE TEMPERED Ξ–HILFER FUZZY FRACTIONAL DIFFERENTIAL EQUATIONS WITH DELAY

RAVICHANDRAN VIVEK, DEVARAJ VIVEK*,
KUPPUSAMY KANAGARAJAN AND ELSAYED M. ELSAYED

Abstract. In the present paper, we investigate the existence and uniqueness of solutions and derive the Ulam-Hyers type stability results for impulsive tempered Ξ-Hilfer fuzzy fractional differential equations with delay. The Banach contraction principle and a Gronwall inequality involving tempered Ξ-Riemann-Liouville fuzzy fractional integral are used. In addition, we offer three examples to clarify the results.

Mathematics subject classification (2020): 26A33, 34A08, 47D09, 47H10, 93C43.
Keywords and phrases: Tempered Ξ-Hilfer fractional derivative, fuzzy differential equations, impulses, stability, delay.

REFERENCES

[1] M. S. ABDO, S. K. PANCHAL AND H. A. WAHASH, *Ulam-Hyers-Mittag-Leffler stability for a ψ-Hilfer problem with fractional order and infinite delay*, Results Appl. Math., **7**, (2020), 100115.
[2] N. AHMAD, A. ULLAH, S. AHMAD, K. SHAH AND I. AHMAD, *On analysis of the fuzzy fractional order Volterra-Fredholm integro-differential equation*, Alex. Eng. J., **60**, 1 (2021), 1827–1838.
[3] A. AHMADIAN, S. SALAHSHOUR AND C. S. CHAN, *Fractional differential system: a fuzzy solution based on operational matrix of shifted Chebyshev polynomials and its applications*, IEEE Trans. Fuzzy Syst., **25**, 1 (2017), 218–236.
[4] R. ALMEIDA, N. R. O. BASTOS AND M. T. T. MONTEIRO, *Modelling some real phenomena by fractional differential equations*, Math. Methods. Appl. Sci., **39**, 16 (2016), 4846–4855.
[5] T. ALLAHVIRANLOO, Z. GOUYANDEH AND A. ARMAND, *Fuzzy fractional differential equations under generalized fuzzy Caputo derivative*, J. Intell. Fuzzy Syst., **26**, (2014), 1481–1490.
[6] E. ARARRABI, M. ELOMARI, S. MELLIANI AND L. S. CHADLI, *Existence and finite-time stability results for a class of nonlinear Hilfer fuzzy fractional differential equations with time-delays*, Filomat., **38**, 8 (2024), 2877–2887.
[7] R. L. BAGLEY AND P. J. TORVIK, *On the appearance of fractional derivatives in the behaviour of real materials*, J. Appl. Mech., **51**, 2 (1984), 294–298.
[8] B. BEDE AND S. G. GAL, *Generalizations of the differentiability of fuzzy-valued functions with applications to fuzzy differential equations*, Fuzzy Sets Syst., **151**, 3 (2005), 581–599.
[9] B. BEDE AND L. STEFANINI, *Generalized differentiability of fuzzy-valued functions*, Fuzzy Set Syst., **230**, (2013), 119–141.
[10] X. CHEN, H. GU AND X. WANG, *Existence and uniqueness for fuzzy differential equation with Hilfer-Katugampola fractional derivative*, Adv. Differ. Equ., 2020, 241.
[11] H. M. FAHAD, A. FERNANDEZ, M. UR REHMAN AND M. SIDDIQI, *Tempered and Hadamard-type fractional calculus with respect to functions*, Mediterr. J. Math., **18**, 4 (2021), 143.
[12] A. GRANAS AND J. DUGUNDJI, *Fixed Point Theory*, New York: Springer, 2003.
[13] J. K. HALE, *Theory of Functional Differential Equations*, Springer, New York, 1977.
[14] R. HILFER, *Applications of Fractional Calculus in Physics*, World Scientific, Singapore, 2000.
[15] N. V. HOA, *Fuzzy fractional functional integral and differential equations*, Fuzzy Sets. Syst., **280**, (2015), 58–90.

Paper DEA-16-17

[16] N. V. HOA AND V. HO, *A survey on the initial value problems of fuzzy implicit fractional differential equations*, Fuzzy Sets. Syst., **400**, (2020), 90–133.

[17] N. V. HOA, V. LUPULESCU AND D. O'REGAN, *A note on initial value problems for fractional fuzzy differential equations*, Fuzzy Sets Syst. **347**, (2018), 54–69.

[18] N. V. HOA, H. VU AND T. M. DUC, *Fuzzy fractional differential equations under Caputo-Katugampola fractional derivative approach*, Fuzzy Sets Syst., **375**, (2019), 70–99.

[19] N. V. HOA, *On the initial value problem for fuzzy differential equations of non-integer order* $\alpha \in (1,2)$, Soft Comput., **24**, 2 (2020), 935–954.

[20] N. V. HOA, P. VAN TRI, T. T. DAO AND I. ZELINKA, *Some global existence results and stability theorem for fuzzy functional differential equations*, J. Intell. Fuzzy Syst., **28**, 1 (2015), 393–409.

[21] K. KANAGARAJAN, R. VIVEK, D. VIVEK AND E. M. ELSAYED, *Existence Results for Fuzzy Differential Equations with* ψ*-Hilfer Fractional Derivative*, Ann. Commun. Math., **5**, 1 (2022), 38–54.

[22] J. P. KHARADE AND K. D. KUCCHE, *On the impulsive implicit* ψ*-Hilfer fractional differential equations with delay*, Math. Methods Appl. Sci. **43**, 4 (2020), 1938–1952.

[23] A. KHASTAN, J. J. NIETO, R. RODRIGUEZ-LOPEZ, *Fuzzy delay differential equations under generalized differentiability*, Inf. Sci., **275**, (2014), 145–167.

[24] A. A. KILBAS, H. M. SRIVASTAVA, J. J. TRJILLO, *Theory and Applications of Fractional Differential Equations*, North-Holland Mathematics Studies, Amsterdam: Elsevier, 2006.

[25] Y. KUANG, *Delay Differential Equations with Applications in Population Dynamics*, Academic Press, Boston, 1993.

[26] K. K. KUCCHE, A. D. MALI. A. FERNANDEZ, H. M. FAHAD, *On tempered Hilfer fractional derivatives with respect to functions and the associated fractional differential equations*, Chaos. Solitons. Fractals., **163**, (2022), 112547.

[27] R. LEWANDOWSKI AND B. CHORAZYCZEWSKI, *Identification of the parameters of the Kelvin-Voigt and the Maxwell fractional models, used to modeling of viscoelastic dampers*, Comput. Struct., **88**, 1–2 (2010), 1–17.

[28] K. B. LIMA, J. V. C. SOUSA AND E. C. OLIVEIRA, *Existence and uniqueness for* ψ*-Hilfer impulsive fractional differential equations*, Proc. Ser. Braz. Soc Comput. Appl. Math., **7**, 1 (2019), 010381.

[29] K. B. LIMA, J. V. C. SOUSA AND E. C. OLIVEIRA, *Ulam-Hyers type stability for* ψ*-Hilfer fractional differential equations with impulses and delay*, Comput. Appl. Math., **40**, 8 (2021), 293.

[30] K. LIU, J. WANG AND D. O'REGAN, *Ulam-Hyers-Mittag-Leffler stability for* ψ*-Hilfer fractional-order delay differential equations*, Adv. Differ. Equ., **2019**, 50, 2019.

[31] A. LIEMERT AND A. KIENLE, *Fundamental solution of the tempered fractional diffusion equation*, J. Math. Phys. **56** (11) (2015), 113504.

[32] V. LUPULESCU, *On a class of fuzzy functional differential equations*, Fuzzy Sets Syst., **160**, (2009), 1547–1562.

[33] A. D. MALI, K. D. KUCCHE, A. FERNANDEZ AND H. M. FAHAD, *On tempered fractional calculus with respect to functions and the associated fractional differential equations*, Math. Methods Appl. Sci., **45**, (2022), 11134–11157.

[34] M. MAZANDARANI AND M. NAJARIYAN, *Type-2 fuzzy fractional derivatives*, Commun. Nonlinear Sci. Numer. Simul., **19**, 7 (2014), 2354–2372.

[35] M. MEDVED AND E. BRESTOVANSKA, *Differential equations with tempered* ψ*-Caputo fractional derivative*, Math. Model Anal., **26**, 4 (2021), 631–50.

[36] S. MELLIANI, E. ARHRRABI, M. H. ELOMARI AND L. S. CHADLI, *Ulam-Hyers-Rassias stability for fuzzy fractional integro differential equations under Caputo gH-differentiability*, Int. J. Optim. Appl., **1**, (2021), 51–55.

[37] N. NAJAFI AND T. ALLAHVIRNLOO, *Semi-analytical methods for solving fuzzy impulsive fractional differential equations*, J. Intell. Fuzzy Syst., **33**, 6 (2017), 3539–3560.

[38] S. RASHID, F. JARAD AND K. M. ABUALNAJA, *On fuzzy Volterra-Fredholm integro differential equation associated with Hilfer-generalized proportional fractional derivative*, AIMS Math., **6**, (2021), 10920–10946.

[39] R. RIVAZ, O. S. FARD AND T. A. BIDGOLI, *On the existence and uniqueness of solutions for fuzzy fractional differential equations*, Tbil. Math. J., **10**, (2017), 197–205.

[40] S. SAIFULLAD, A. ALI, A. KHAN, K. SHAH AND T. ABDELJAWAD, *A novel tempered fractional transform: Theory, properties and applications to differential equations*, Fractals., 1 2023.

[41] J. V. C. Sousa and E. C. Oliveira, *On the ψ-Hilfer fractional derivative*, Commun. Nonlinear Sci. Numer. Simul., **60**, (2018), 72–91.
[42] R. Vivek, D. Vivek, K. Kanagarajan and E. M. Elsayed, *Existence and finite-time stability of fuzzy delay differential equations with tempered-regularized Hilfer fractional derivative*, Math. Sci. Lett., **11**, 3 (2022), 95–102.
[43] R. Vivek, D. Vivek, K. Kanagarajan and E. M. Elsayed, *Some results on the study of Ξ-Hilfer type fuzzy fractional differential equations with time delay*, Proc. Int. Math. Sci., **4**, 2 (2022), 65–76.
[44] R. Vivek, D. Vivek, K. Kanagarajan and E. M. Elsayed, *Existence and stability of ψ-Hilfer fuzzy fractional differential equations with boundary conditions*, Math. Eng. Aerosp., **14**, 3 (2023), 669–685.
[45] R. Vivek, D. Vivek, K. Kanagarajan and E. M. Elsayed, *Dynamics and stability of Ξ-Hilfer fractional fuzzy differential equations with impulses*, Commun. Adv. Math. Sci., **6**, 3 (2023), 115–127.
[46] H. Vu and N. V. Hoa, *Hyers-Ulam stability of fuzzy fractional Volterra integral equations with the kernel ψ-function via successive approximation method*, Fuzzy Sets Systs. **419**, (2021), 67–98.
[47] F. Yu, *Integrable coupling system of fractional soliton equation hierarchy*, Phys. Lett., **373**, 41 (2009), 3730–3733.
[48] X. Wang, D. Luo and Q. Zhu, *Ulam-Hyers stability of Caputo type fuzzy fractional differential equations with time-delays*, Chaos Solit. Fractals., **156**, (2022), 111822.

Differential Equations & Applications
www.ele-math.com
dea@ele-math.com