# POSITIVE SOLUTIONS FOR A RIEMANN–LIOUVILLE FRACTIONAL SYSTEM WITH $\rho$–LAPLACIAN OPERATORS

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*Abstract.* This paper studies the existence and uniqueness of positive solutions for Riemann-Liouville fractional differential equations with $\rho$-Laplacian operators and coupled nonlocal boundary conditions involving the Riemann-Stieltjes integrals. By means of an interesting fixed point theorem, some new sufficient conditions guaranteeing the existence and uniqueness of positive solutions are presented, and the unique positive solution can be the limit of a sequence constructed for any given initial point in a special set. To demonstrate the conclusion, a good example is given.

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