Differential
Equations
& Applications
Volume 17, Number 1 (2025), 35–52
doi:10.7153/dea-2025-17-03

EXISTENCE AND GLOBAL BEHAVIOR OF POSITIVE SOLUTIONS OF SEMILINEAR FRACTIONAL DIRICHLET PROBLEMS IN EXTERIOR DOMAINS

SANA SALAH*, FATEN TOUMI AND MOHAMED HBAIEB

Abstract. In this paper, we establish the existence and the global asymptotic behavior of positive solutions in an exterior domain $\Omega \subset \mathbb{R}^d$, $d \geqslant 3$,

$$\begin{cases} (-\Delta)^{\frac{\alpha}{2}} x = f(t)x^p, & \text{in } \Omega, \\ x > 0, & \text{in } \Omega, \\ \lim\limits_{t \to \partial\Omega} \delta(t)^{1-\frac{\alpha}{2}} x(t) = 0, \\ \lim\limits_{|t| \to \infty} x(t) = 0, \end{cases}$$

where $(-\Delta)^{\frac{\alpha}{2}}$ is the infinitesimal generator of a killed symmetric α-stable process X^Ω on Ω, $0 < \alpha < 2$, $p < 1$ and the function f is positive and satisfies the suitable conditions related to the Karamata classes $\mathscr{K}_0$ and $\mathscr{K}_\infty$. Our approach relies on potential theory, Karamata regular variation theory, and the Schauder fixed point theorem.

Mathematics subject classification (2020): 34A08, 31B05, 34B27, 34K37, 35R11, 60G22.
Keywords and phrases: Dirichlet problem, fractional Laplacian, Green function, Karamata class, Kato class, Schauder fixed point theorem.

REFERENCES

[1] R. ALSAEDI AND H. MÂAGLI AND N. ZEDDINI, *Estimates on potential functions and boundary behavior of positive solutions for sublinear Dirichlet problems*, Electronic Journal of Differential Equations **08**, (2014), 1–10,
[2] I. BACHAR AND H. MÂAGLI, *Singular solutions of a nonlinear equation in a punctured domain of* $\mathbb{R}^2$, Discrete & Continuous Dynamical Systems-Series S **12**, (2019).
[3] I. BACHAR AND H. MÂAGLI AND S. MESLOUB *Positive solutions of semilinear problems in an exterior domain of* $\mathbb{R}^2$, Boundary Value Problems (2019), 1–24.
[4] J. BLIEDTNER AND W. HANSEN, *Potential theory: An analytic and probabilistic approach to balayage*, Springer Science & Business Media, 2012.
[5] K. BOGDAN AND T. ŻAK, *On Kelvin transformation*, Journal of Theoretical Probability **19**, (2006), 89–120.
[6] R. CHEMMAM AND H. MÂAGLI, *Existence of Positive Solutions for Some Dirichlet Problems Associated to Fractional Laplacian in Exterior Domains*, Communications in Mathematical Analysis **13**, (2012), 27–55.
[7] R. CHEMMAM AND H. MÂAGLI AND S. MASMOUDI AND M. ZRIBI, *Combined effects in nonlinear singular elliptic problems in a bounded domain*, Advances in Nonlinear Analysis **1**, (2012), 301–318.
[8] R. CHEMMAM AND H. MÂAGLI AND S. MASMOUDI, *On a new Kato class and positive solutions of Dirichlet problems for the fractional Laplacian in bounded domains*, Nonlinear Analysis: Theory, Methods & Applications **74**, (2011) 1555–1576.

Paper DEA-17-03

[9] R. CHEMMAM AND H. MÂAGLI AND S. MASMOUDI, *Boundary behavior of positive solutions of a semilinear fractional Dirichlet problem*, Journal of Abstract Differential Equations and Applications **3**, (2012), 75–90.
[10] R. CHEMMAM AND A. DHIFLI AND H. MÂAGLI, *Asymptotic behavior of ground state solutions for sublinear and singular nonlinear Dirichlet problems*, Electron. J. Differ. Equ. **88**, (2011), 1–12.
[11] Z. Q. CHEN AND R. SONG, *Estimates on Green functions and Poisson kernels for symmetric stable processes*, Mathematische Annalen **312**, (1998), 465–501.
[12] A. DHIFLI AND H. MÂAGLI AND M. ZRIBI, *On the subordinate killed BM in bounded domains and existence results for nonlinear fractional Dirichlet problems*, Mathematische Annalen **352**, (2012), 259–291.
[13] H. MÂAGLI, *Invariance Des Fonctions-Harmoniques Par Les Trans-Formations De Moebius*, Expositiones Mathematicae **13**, (1995) 469–480.
[14] H. MÂAGLI AND M. ZRIBI, *On a semilinear fractional Dirichlet problem on a bounded domain*, Appl. Math. Comput. **222**, (2013) 331–342.
[15] H. MÂAGLI AND S. TURKI AND Z. ZINE EL ABIDINE, *Asymptotic behavior of positive solutions of a semi-linear Dirichlet problem outside the unit ball*, Electronic Journal of Differential Equations **95**, (2013), 1–14.
[16] H. MÂAGLI AND A. K. ALZAHRANI AND Z. ZINE EL ABIDINE, *Asymptotic behavior of positive solutions of a semilinear Dirichlet problem in exterior domains*, Electronic Journal of Differential Equations **137**, (2018) 1–14.
[17] V. MARIĆ, *Regular Variation and Differential Equations*, Lecture notes in mathematics, 2000.
[18] H. P. MCKEAN, *Sample Functions of Stable Processes*, Annals of Mathematics **61**, 1955.
[19] S. SALAH AND F. TOUMI AND M. HBAIB, *Singular solutions of a fractional Dirichlet problem in a punctured domain*, Filomat **38**, no. 2, (2024), 449–472.
[20] S. SALAH AND F. TOUMI AND M. HBAIB, *New results for singular solutions for semilinear fractional Dirichlet problems on punctured domains*, submitted, 1–18.
[21] E. SENETA, *Regularly varying functions*, Lecture Notes in Math. **508**, 1976.