NEUMANN PROBLEM FOR A STOCHASTIC BENJAMIN–BONA–MAHONY EQUATION WITH RIESZ FRACTIONAL DERIVATIVE

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Abstract. In this work, we study an initial boundary-value problem for a stochastic Benjamin-Bona-Mahony equation with Riesz-fractional spatial derivative and white noise on the half-line. For the associated linear problem, we construct the Green's function adapting the main ideas of the Fokas method. Then, the main problem will be understood in the Walsh sense and the Picard scheme is used to prove existence and uniqueness of solutions. Moreover, an example is presented to show the results obtained.

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