

ON CONTROLLABILITY OF SEMILINEAR GENERALIZED IMPULSIVE SYSTEMS ON FINITE DIMENSIONAL SPACE

VISHANT SHAH, GARGI TRIVEDI*, JAITA SHARMA,
PRAKASHKUMAR H. PATEL AND NEETA CHUDASAMA

Abstract. This article presents sufficient conditions for the complete controllability of generalized semilinear impulsive systems in a finite-dimensional space. The analysis focuses on cases where the nonlinear perturbation functions satisfy the Lipschitz continuity condition. We establish these conditions by leveraging functional analysis techniques and various fixed-point theorems. Furthermore, a numerical example is included to demonstrate the effectiveness of the proposed results.

Mathematics subject classification (2020): 93B05, 34H05, 93C10, 93C15.

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