Differential
Equations
& Applications
Volume 17, Number 2 (2025), 85–105
doi:10.7153/dea-2025-17-06

SOME PROPERTIES OF LOGARITHMIC p–LAPLACE OPERATORS AND APPLICATIONS

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Abstract. A study of the existence and uniqueness of the weak solution to a class of nonlinear elliptic equations governed by the logarithmic perturbation is offered. We exploit interesting properties of the new modular function involving $L^p\log^\alpha L$-growth and the optimal embedding theorem for Orlicz-Sobolev spaces. We are concerned with these properties in analyzing the existence and uniqueness of the solution by the theory of pseudo-monotone operators proposed in [2, 3] combined with variational methods. Our approach deals not only with problems of the p-Laplacian type but also yields a slight extension of the results for more general differential operators with a similar structure.

Mathematics subject classification (2020): 35D30, 35J92, 46E30.

Keywords and phrases: Existence result, logarithmic perturbation, $L^p\log^\alpha L$-operators, pseudo-monotone operators, Orlicz-Sobolev spaces.

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