

A NOTE ON THE BESSEL DIFFERENTIAL EQUATION

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Abstract. In this paper, we will show the general solution of the Bessel differential equation given by $x^2 y'' + xy' + (x^2 - \nu^2)y = 0$, where $\nu, x \in \mathbb{R}$ and $x > 0$, but only when $\nu = \frac{2m-1}{2}$ with $m \in \mathbb{N}$. Moreover, contrary to what we found in the literature, our general solution does not depend on a series of functions, our algorithm provides the exact general solution.

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