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EXISTENCE OF SOLUTION AND ASYMPTOTIC BEHAVIOR FOR THE NAVIER–STOKES EQUATIONS WITH GENERAL DAMPING

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Abstract. We establish the existence of solution for the Navier-Stokes equations with a general damping term, we give examples. We also derive estimates on the asymptotic behavior of the solution, for instance, energy estimate decay in time and extinction in time. We construct a sequence of solutions of auxiliary equations in finite dimension that converges to a genuine solution of the original equations.

Mathematics subject classification (2020): 35Q30, 35Q35, 65N30, 76N10.
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