Differential
Equations
& Applications
Volume 17, Number 3 (2025), 133–162
doi:10.7153/dea-2025-17-09

EIGENVALUE PROBLEMS FOR A COUPLED SYSTEM OF SINGULAR KATUGAMPOLA FRACTIONAL DIFFERENTIAL EQUATIONS WITH FOUR–POINT BOUNDARY CONDITIONS

WENGUI YANG

Abstract. This paper investigates the nonlinear eigenvalue problems (NEPs) for a class of singular Katugampola fractional differential systems with four-point coupled boundary conditions. Firstly, we establish the Green's functions of the aforementioned NEPs and their fundamental analytic properties. Secondly, utilizing some classical fixed-point theorems, some explicit eigenvalue interval-dependent criteria are derived for the existence of at least one positive solution to the addressed NEPs. As applications, some examples are presented to illustrate the feasibility and effectiveness of our main results.

Mathematics subject classification (2020): 34A08, 34B16, 34B18.
Keywords and phrases: Katugampola fractional differential equations, eigenvalue problems, four-point boundary conditions, Green's functions, Krasnoselskii's fixed-point theorem.

REFERENCES

[1] B. AHMAD, S. K. NTOUYAS, *Nonlinear systems of fractional differential equations*, Springer Cham, 2024.
[2] B. BASTI, Y. ARIOUA, N. BENHAMIDOUCHE, *Existence and uniqueness of solutions for nonlinear Katugampola fractional differential equations*, J. Math. Appl., **42** (2019), 35–61.
[3] S. BATIK, F. Y. DEREN, *Semipositone fractional boundary value problems with n point fractional integral boundary conditions*, Miskolc Math. Notes, **23** (1) (2022), 93–104.
[4] A. BOUCHRA, B. TELLAB, K. ZENNIR, *Positive solutions for integral nonlinear boundary value problem in fractional Sobolev spaces*, Math. Meth. Appl. Sci., **46** (3) (2023), 3115–3131.
[5] H. DJOURDEM, S. BENAICHA, *Positive solutions for fractional differential equations with non-separated type nonlocal multi-point and multi-term integral boundary conditions*, Studia Univer. Babes-Bolyai Mat., **66** (4) (2021), 691–708.
[6] D. GUO, V. LAKSHMIKANTHAM, *Nonlinear problems in abstract cones*, New York: Academic Press, 1988.
[7] X. HAO, L. ZHANG, L. LIU, *Positive solutions of higher order fractional integral boundary value problem with a parameter*, Nonlinear Anal. Model. Control, **24** (2) (2019), 210–223.
[8] J. HENDERSON, R. LUCA, A. TUDORACHE, *On a system of Riemann-Liouville fractional boundary value problems with ρ-laplacian operators and positive parameters*, Fractal Fract., **6** (6) (2022), 299.
[9] F. JARAD, T. ABDELJAWAD, D. BALEANU, *On the generalized fractional derivatives and their Caputo modification*, J. Nonlinear Sci. Appl., **10** (2017), 2607–2619.
[10] J. JIANG, L. LIU, Y. WU, *Positive solutions to singular fractional differential system with coupled boundary conditions*, Commun. Nonlinear Sci. Numer. Simul., **18** (11) (2013), 3061–3074.
[11] U. N. KATUGAMPOLA, *New approach to a generalized fractional integral*, Appl. Math. Comput., **218** (2011), 860–865.
[12] U. N. KATUGAMPOLA, *A new approach to generalized fractional derivatives*, Bull. Math. Anal. Appl., **6** (2014), 1–15.
[13] Y. LI, W. CHENG, J. XU, *Monotone iterative schemes for positive solutions of a fractional differential system with integral boundary conditions on an infinite interval*, Filomat, **34** (13) (2020), 4399–4417.

Paper DEA-17-09

[14] R. LUCA, A. TUDORACHE, *On a system of Hadamard fractional differential equations with nonlocal boundary conditions on an infinite interval*, Fractal Fract., **7** (6) (2023), 458.
[15] B. ŁUPIŃSKA, T. ODZIJEWICZ, *A Lyapunov-type inequality with the Katugampola fractional derivative*, Math. Meth. Appl. Sci., **41** (2018), 8985–8996.
[16] B. ŁUPIŃSKA, E. SCHMEIDEL, *Analysis of some Katugampola fractional differential equations with fractional boundary conditions*, Math. Biosci. Eng., **18** (6) (2021), 7269–7279.
[17] K. NEAMPREM, T. MUENSAWAT, S. K. NTOUYAS, J. TARIBOON, *Positive solutions for fractional differential systems with nonlocal Riemann-Liouville fractional integral boundary conditions*, Positivity, **21** (3) (2016), 825–845.
[18] S. N. RAO, *Multiplicity of positive solutions for fractional differential equation with p-Laplacian boundary value problems*, Int. J. Differ. Equ., **2016** (2016), 6906049.
[19] A. SOUAHI, A. GUEZANE-LAKOUD, A. HITTA, *Positive Solutions for Higher-Order Nonlinear Fractional Differential Equations*, Vietnam J. Math., **45** (3) (2016), 441–450.
[20] S. N. SRIVASTAVA, S. PATI, J. R. GRAEF, A. DOMOSHNITSKY, S. PADHI, *Existence of solution for a Katugampola fractional differential equation using coincidence degree theory*, Mediterr. J. Math., **21** (2024), 123.
[21] M. SUBRAMANIAN, K. ABUASBEH, M. MANIGANDAN, *On the generalized Liouville-Caputo type fractional differential equations supplemented with Katugampola integral boundary conditions*, Symmetry, **14** (2022), 2273.
[22] J. TARIBOON, S.K. NTOUYAS, W. SUDSUTAD, *Positive solutions for fractional differential equations with three-point multi-term fractional integral boundary conditions*, Adv. Differ. Equ., **2014** (2014), 28.
[23] A. TUDORACHE, R. LUCA, *Positive solutions for a fractional differential equation with sequential derivatives and nonlocal boundary conditions*, Symmetry, **14** (9) (2022), 1779.
[24] Y. WANG, L. LIU, Y. WU, *Positive solutions for a class of higher-order singular semipositone fractional differential systems with coupled integral boundary conditions and parameters*, Adv. Differ. Equ., 2014 (2014), 268.
[25] W. YANG, *Monotone iterative technique for a coupled system of nonlinear Hadamard fractional differential equations*, J. Appl. Math. Comput., **59** (1) (2019), 585–596.
[26] W. YANG, *Positive solutions for a singular coupled system of nonlinear higher-order fractional q-difference boundary value problems with two parameters*, Differ. Equ. Appl., **11** (4) (2019), 509–529.
[27] W. YANG, *Positive solutions for a class of nonlinear p-Laplacian Hadamard fractional differential systems with coupled nonlocal Riemann-Stieltjes integral boundary conditions*, Filomat, **36** (19) (2022), 6631–6654,
[28] H. ZHANG, Z. HAO, M. BOHNER, *Multiple and nonexistence of positive solutions for a class of fractional differential equations with p-Laplacian operator*, Mathematics, **12** (23) (2024), 3869.
[29] W. ZHANG, C. ZHAI, *Positive solutions for a Riemann-Liouville fractional system with ρ-Laplacian operators*, Differ. Equ. Appl., **16** (4) (2024), 305–319.