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DYNAMICS AND STABILITY OF STOCHASTIC PANTOGRAPH DIFFERENTIAL EQUATION WITH COMPOSITE FRACTIONAL DERIVATIVE

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Abstract. In this paper, we study the existence and uniqueness of solutions, as well as the Ulam-Hyers and Ulam-Hyers-Rassias stability, for a class of stochastic pantograph differential equations involving the Hilfer fractional derivative. The analysis is carried out using the Picard operator theory. Furthermore, a numerical example is provided to illustrate and validate the theoretical results.

Mathematics subject classification (2020): 26A33, 34K50, 34A12, 34D20.
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Paper DEA-18-01

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