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EXISTENCE AND UNIQUENESS OF SOLUTIONS FOR NONLINEAR FRACTIONAL CAPUTO–LANGEVIN EQUATION WITH ANTI–PERIODIC BOUNDARY VALUE PROBLEM

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Abstract. In this paper, we consider a nonlinear fractional Caputo-Langevin equation with anti-periodic boundary value problem. Existence results and uniqueness are obtained by using Schauder fixed point theorem and Banach contraction mapping principle. As applications, two examples are offered to illustrate our main results.

Mathematics subject classification (2020): 26A33, 39A13, 34B15.
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