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EXISTENCE AND NONEXISTENCE OF SIGN–CHANGING SOLUTIONS FOR LINEARLY PERTURBED SUPERLINEAR EQUATIONS ON EXTERIOR DOMAINS

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Abstract. In this paper, we study the radial solutions of $\Delta u + K(|x|)f(u) + (N-2)^2 u/|x|^{2+(N-2)\delta} = 0$, $0 < \delta < 2$ in the exterior of the ball of radius $R > 0$ in $\mathbb{R}^N$ where f grows superlinearly at infinity and is singular at zero with $f(u) \sim -1/|u|^{q-1}u$ and $0 < q < 1$ for small u. We assume $K(|x|) \sim |x|^{-\alpha}$ for large $|x|$ and establish the existence of an infinite number of sign-changing solutions when $N + q(N-2) < \alpha < 2(N-1)$. We also prove nonexistence for $0 < \alpha \leqslant 2$.

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