Differential
Equations
& Applications
Volume 18, Number 2 (2026), 139–146
doi:10.7153/dea-2026-18-09

IMPROVED OSCILLATION CRITERIA OF GENERAL CAUCHY—EULER TYPE FOURTH–ORDER DELAY DIFFERENTIAL EQUATIONS

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Abstract. This paper addresses the fourth-order strongly noncanonical delay differential equation of the form

$$\Big(p_3(t)\Big(p_2(t)\Big(p_1(t)x'(t)\Big)'\Big)'\Big)'+q(t)x(\tau(t))=0$$

with

$$\int_{t_0}^{\infty}\frac{1}{p_j(t)}dt<\infty,\quad j=1,2,3.$$

Employing a robust analytical framework, we develop improved oscillation criteria inspired by traditional approaches like those of Hille and Nehari. By transforming noncanonical forms into the corresponding canonical equivalents, we establish conditions that govern the oscillatory nature of equations of this type more effectively. The improvement of the new result over the known ones is demonstrated via an example.

Mathematics subject classification (2020): 34C10, 34K11.
Keywords and phrases: Oscillation, noncanonical, fourth-order, delay differential equation.

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Paper DEA-18-09

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