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MATHEMATICAL MODELING OF ANOMALOUS DIFFUSION IN POROUS MEDIA

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Abstract. Analysis of diffusion in a complex environment shows that the conventional diffusion equation based on Fick's law fails to model the anomalous character of the diffusive mass transport observed in the field and laboratory experiments. New mathematical models of diffusive transport, different from Fick's law, were proposed and validated in literature. In the present paper the examples of the equations that can be used for describing the anomalous mass transport are presented and some important properties of these equations are discussed. Two regimes of anomalous diffusion are identified. One regime, which is called sub-diffusion, is characterized by the slower propagation of the concentration front, so that the squared distance of the front passage requires longer time than in the case of the classical Fickian diffusion. The second regime (called super-diffusion) is characterized by the higher diffusion rate, so that the particles will pass the specified distance faster than in the case of classical Fickian diffusion. Both regimes can be modeled by non-local diffusion equation with temporal and spatial fractional derivatives. It is shown that equation with spatially variable diffusivity proposed by O'Shaughnessy and Procaccia (1985), which provides a relatively good model of diffusion on a regular fractal, is less applicable for describing the effects of sub and super diffusion that may take place in a fractured porous medium or any other complex medium.

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