

EXISTENCE AND UNIQUENESS OF SOLUTIONS FOR NONLINEAR FRACTIONAL DIFFERENTIAL EQUATIONS WITH INTEGRAL BOUNDARY CONDITIONS

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Abstract. In this paper, we study existence and uniqueness of solutions to nonlinear fractional differential equations with integral boundary conditions in an ordered Banach space. We use the Caputo fractional differential operator and the nonlinearity depends on the fractional derivative of an unknown function. For the existence of solutions, we employ the nonlinear alternative of Leray-Schauder and the Banach fixed point theorem. An example is included to show the applicability of our results.

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