

THE FRACTIONAL VIRIAL THEOREM

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Abstract. Fractional calculus is an emerging field and its has many applications in several fields of science and engineering. One of the major issue in this field is to apply this type of calculus to the real world applications. In this paper the fractional generalization of the classical virial theorem is presented.

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