

## A STUDY OF IMPULSIVE FRACTIONAL DIFFERENTIAL INCLUSIONS WITH ANTI-PERIODIC BOUNDARY CONDITIONS

BASHIR AHMAD AND JUAN J. NIETO

*Abstract.* In this paper, we prove the existence of solutions for impulsive fractional differential inclusions with anti-periodic boundary conditions by applying Bohnenblust-Karlin's fixed point theorem.

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