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ON THE STABILITY ANALYSIS OF WEIGHTED AVERAGE FINITE DIFFERENCE METHODS FOR FRACTIONAL WAVE EQUATION

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Abstract. In this article, a numerical study for the fractional wave equations is introduced by using a class of finite difference methods. These methods are extension of the weighted average methods for ordinary (non-fractional) wave equations. The stability analysis of the proposed methods is given by a recently proposed procedure similar to the standard John von Neumann stability analysis. Simple and accurate stability criterion valid for different discretization schemes of the fractional derivative, arbitrary weight factor, and arbitrary order of the fractional derivative, is given and checked numerically. Numerical test example and comparisons have been presented for clarity.

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