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APPLICATION OF THE MIXED MONOTONE OPERATOR METHOD TO FRACTIONAL BOUNDARY VALUE PROBLEMS

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Abstract. The authors use the mixed monotone operator method to study the fractional boundary value problem

$$-D_{0+}^{\nu} u(t) = \lambda\, f(t,u),\ t \in (0,1),$$
$$u^{(j)}(0) = 0,\ j = 0,\dots,n-2, \quad [D_{0+}^{\alpha} u(t)]_{t=1} = 0.$$

Here, $m \geqslant 1$ and $n \geqslant 3$ are integers, $n-1 < \nu \leqslant n$, $1 \leqslant \alpha \leqslant n-2$, $u(t) = (u_1(t),\dots,u_m(t))^T$, $\lambda = (\lambda_1,\dots,\lambda_m)$, $\lambda\, f(t,u) = (\lambda_1 f_1(t,u),\dots,\lambda_m f_m(t,u))^T$, and D_{0+}^{ν} is the Riemann-Liouville fractional derivative of order ν. Existence, uniqueness, and dependence of positive solutions on the parameter λ are discussed. An application to a special problem is also presented.

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