Fractional Differential Calculus
Volume 2, Number 2 (2012), 119–128
doi:10.7153/fdc-02-09

MULTIPLE POSITIVE SOLUTIONS FOR NONLINEAR FRACTIONAL DIFFERENTIAL SYSTEMS

NEMAT NYAMORADI AND TAHEREH BASHIRI

Abstract. In this paper, we study the existence of positive solutions to boundary value problem for fractional differential system

$$\begin{cases} D^{\alpha}_{0^+}u(t)+f_1(t,u(t),v(t))=0, & t\in(0,1),\\ D^{\alpha}_{0^+}v(t)+f_2(t,u(t),v(t))=0, & t\in(0,1),\ 1<\alpha\leqslant 2,\\ u(0)=0,\quad D^{\beta}_{0^+}u(1)-\mu_1 D^{\beta}_{0^+}u(\eta_1)=\lambda_1,\\ v(0)=0,\quad D^{\beta}_{0^+}v(1)-\mu_2 D^{\beta}_{0^+}v(\eta_2)=\lambda_2,\ 0<\beta<1, \end{cases}$$

where $D^{\alpha}_{0^+}$ is the Riemann-Liouville fractional derivative of order α. By using the Leggett-Williams fixed point theorem in a cone, the existence of three positive solutions for nonlinear singular boundary value problems is obtained.

Mathematics subject classification (2010): 34K15.

Keywords and phrases: Cone, multi point boundary value problem, fixed point theorem, Riemann-Liouville fractional derivative.

REFERENCES

[1] A. M. A. EL-SAYED, *Nonlinear functional differential equations of arbitrary orders*, Nonlinear Anal. 33 (1998) 181–186.

[2] A. A. KILBAS, J. J. TRUJILLO, *Differential equations of fractional order: Methods, results and problems I*, Appl. Anal. 78 (2001) 153–192.

[3] A. A. KILBAS, J. J. TRUJILLO, *Differential equations of fractional order: Methods, results and problems II*, Appl. Anal. 81 (2002) 435–493.

[4] G. SAMKO, A. KILBAS, O. MARICHEV, *Fractional Integrals and Derivatives: Theory and Applications*, Gordon and Breach, Amsterdam, 1993.

[5] I. PODLUBNY, *Fractional Differential Equations*, Academic Press, New York, 1999.

[6] R. P. AGARWAL, M. BENCHOHRA, S. HAMANI, *A survey on existence results for boundary value problems of nonlinear fractional differential equations and inclusions*, Acta Appl. Math. 109 (2010) 973–1033.

[7] V. LAKSHMIKANTHAM, A. S. VATSALA, *Basic theory of fractional differential equations*, Nonlinear Anal. TMA 69 (8) (2008) 2677–2682.

[8] A. SAADI, M. BENBACHIR, *Positive solutions for three-point nonlinear fractional boundary value problems*, Electron. J. Qual. Theory Differ. Equ. 2 (2011) 1–19.

[9] C. F. LI, X. N. LUO, Y. ZHOU, *Existence of positive solutions of the boundary value problem for nonlinear fractional differential equations*, Comput. Math. Apll. 59 (2010) 1363–1375.

[10] A. A. KILBAS, H. M. SRIVASTAVA, J. J. TRUJILLO, *Theory and Applications of Fractional Differential Equations*, Elsevier, Amsterdam, 2006.

[11] R. W. LEGGETT, L. R. WILLIAMS, *Multiple positive fixed points of nonlinear operators on ordered Banach spaces*, Indiana Univ. Math. J. 28 (1979) 673–688.